



QP – 232

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**I Semester B.A./B.Sc. Examination, April/May 2021**  
**(CBCS) (2020 – 21 and Onwards) (Fresh)**  
**Paper – I : MATHEMATICS**

Time : 3 Hours

Max. Marks : 70

**Instruction : Answer all questions.**

PART – A



I. Answer any five questions.

(5×2=10)

a) Reduce the matrix  $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$  to echelon form.

b) Find the eigen values of the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .

c) Find the  $n^{\text{th}}$  derivative of  $\sin^2 x$ .

d) If  $z = x^3 - 4x^2y + 5y^2$ , find  $\frac{\partial^2 z}{\partial x \partial y}$ .

e) Evaluate  $\int_0^{\pi/2} \cos^8 x \, dx$ .

f) Evaluate  $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$ .

g) Find the equation of the sphere whose centre is  $(3, -1, 4)$  and which passes through the point  $(1, -2, 0)$ .

h) Show that the spheres  $x^2 + y^2 + z^2 + 6y + 14z + 28 = 0$  and  $x^2 + y^2 + z^2 + 6x + 8y + 4z + 24 = 0$  cut orthogonally.

PART – B

II. Answer any three questions.

(5×3=15)

a) Find the rank of the matrix  $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$  by reducing it to normal form.

P.T.O.



b) Solve the system of equations :

$$x_1 + 3x_2 + 2x_3 = 0$$

$$2x_1 - x_2 + 3x_3 = 0$$

$$3x_1 - 5x_2 + 4x_3 = 0$$

$$x_1 + 17x_2 + 4x_3 = 0.$$

c) Show that the equations  $x + y + z = 6$ ,  $x + 2y + 3z = 14$ ,  $x + 4y + 7z = 30$  are consistent and solve them.

d) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$ .

e) Using Cayley Hamilton theorem, find the inverse of the matrix  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ .

### PART – C

III. Answer **any three** questions.

(5×3=15)

a) Find the  $n^{\text{th}}$  derivative of  $\frac{x+3}{(x-1)(x+2)}$ .

b) If  $x = \sin t$  and  $y = \cos pt$  then prove that  $(1 - x^2) y_{n+2} - (2n+1)xy_{n+1} - (n^2 - p^2)y_n = 0$ .

c) State and prove Euler's theorem for homogeneous function of two variables.

d) If  $z = \sin\left(\frac{x}{y}\right)$  and  $x^2 + y^2 = a^2$ , find  $\frac{dz}{dx}$ .

e) If  $u = x^2 + y^2 + z^2$ ,  $v = xy + yz + zx$ ,  $w = x + y + z$ . Find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .

### PART – D

IV. Answer **any two** questions.

(5×2=10)

a) Obtain the reduction formula for  $\int \sec^n x \, dx$ , where  $n$  is a positive integer.

b) Evaluate  $\int_0^\pi x \sin^6 x \, dx$ .

c) Evaluate  $\int_0^1 \frac{x^a - 1}{\log x} \, dx$ , where  $a$  is a parameter, using differentiation under integral sign.



PART – E

V. Answer **any two** questions.

(5×2=10)

- a) Find the equation of the tangent plane to the sphere,  $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$  at  $(1, 2, 3)$ .
- b) Find the equation of the right circular cone generated by revolving the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  about the line  $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$ .
- c) Find the equation of a right circular cylinder of radius 2 whose axis passes through  $(1, 2, 3)$  and has the direction cosines proportional to  $2, -3, 6$ .

PART – F

VI. Answer **any two** questions.

(5×2=10)

- a) If  $y = (t^2 + 1)^2$  is the displacement of a particle. Find its velocity and acceleration at  $t = 2$ .
- b) If 15 kgs of commodity A and 17 kgs of commodity B together cost Rs. 241 and 25 kgs of commodity A and 13 kgs of commodity B together costs Rs. 279. Find the price of each per kg by using row reduced echelon form.
- c) If a metallic ball of radius 3 units is enclosed within a cylindrical pipe. Find the equation of the cylinder, if its axis is  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ .